Quiz 11

October 7, 2016

Show all work and circle your final answer.

1. Compute $f'(x)$ if $f(x) = \ln(\ln(3\sqrt{x}))$.

can rewrite as $\ln(3)+\frac{1}{2}\ln x$

$$f'(x) = \frac{1}{\ln(3x^{1/2})} \cdot \frac{d}{dx}\left(\ln(3x^{1/2})\right)$$

$$= \frac{1}{\ln(3x^{1/2})} \cdot \frac{1}{3x^{1/2}} \cdot \frac{d}{dx}(3x^{1/2})$$

$$= \frac{1}{\ln(3x^{1/2})} \cdot \frac{1}{\cancel{3}x^{1/2}} \cdot \frac{\cancel{3}}{2x^{1/2}} = \boxed{\frac{1}{2x\ln(3x^{1/2})}}$$

2. Evaluate $\lim_{h \to 0} \frac{\ln((h+1)^2)}{h}$.

Notice $\lim_{h\to 0} \frac{\ln((h+1)^2)}{h} = f'(1)$ where $f(x) = \ln(x^2)$.

So $\lim_{h\to 0} \frac{\ln((h+1)^2)}{h} = \frac{d}{dx}\left[\ln(x^2)\right]$ at $x=1$

$= \frac{d}{dx}\left[2\ln x\right]$ at $x=1$

$= \frac{2}{1} = \boxed{2}$

3. Find $\frac{dy}{dx}$ if $y = x^{\sin x}$.

Use logarithmic differentiation.

$\ln y = \ln(x^{\sin x})$

$\ln y = (\sin x)(\ln x)$

$\frac{1}{y}y' = \cos x(\ln x) + \sin x\left(\frac{1}{x}\right)$

$$\boxed{y' = x^{\sin x}\left[\cos x \ln x + \frac{\sin x}{x}\right]}$$